AP® Calculus BC

2003 Sample Student Responses

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1 1 1 1 1 1 1 1 1 1 C1

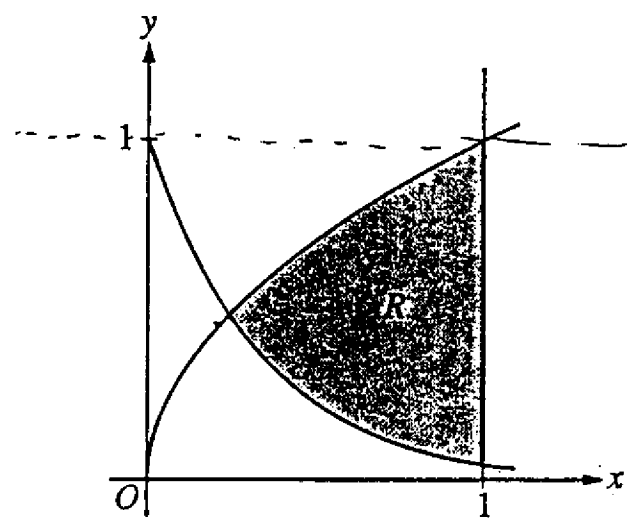
CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



line of rotation for part B

Work for problem 1(a)

area of $R = A(R) = \int_{.239}^{1} \sqrt{x} - e^{-3x}\,dx$

$A(R) = .443$

intersect of $y=\sqrt{x}$ & $y=e^{-3x}$ is the lower bound for the integral!

$\sqrt{x} = e^{-3x}$

$x = .239$

Continue problem 1 on page 5.

Work for problem 1(b)

Volume of solid $= \pi \int_{.239}^{1} R^2 - r^2 \, dx$ $\quad R = 1 - e^{-3x}$ $\quad r = 1 - \sqrt{x}$

or

$$V = \pi \int_{.239}^{1} (1 - e^{-3x})^2 - (1 - \sqrt{x})^2 \, dx$$

$V = 1.424$

Work for problem 1(c)

$V = \int_{.239}^{1} h \cdot b \, dx$ $\quad h = 5b$ $\quad b = \sqrt{x} - e^{-3x}$

$h = 5(\sqrt{x} - e^{-3x})$

$$V = \int_{.239}^{1} 5(\sqrt{x} - e^{-3x})(\sqrt{x} - e^{-3x}) \, dx$$

$V = 1.554$

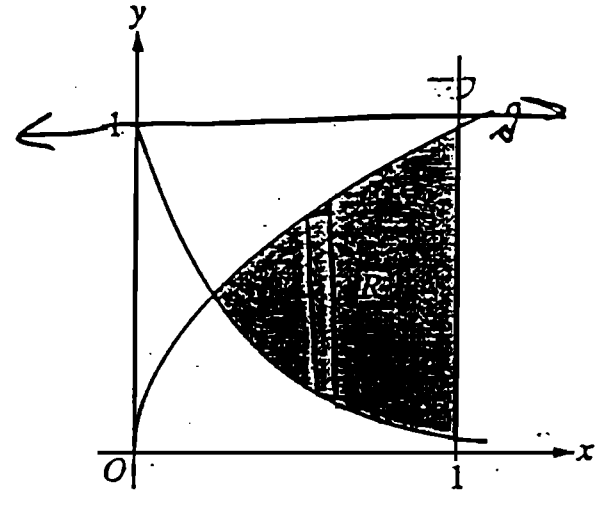
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$$\text{Area} = \int_{.24}^{1} \sqrt{x} - e^{-3x}\, dx = .443 \text{ units}^2$$

Finding bounds: $\sqrt{x} = e^{-3x}$

$x \approx .24$

Continue problem 1 on page 5.

1 1 1 1 1 1 1 1 1 1 F_2

Work for problem 1(b)

Found volume using Washer method $V = \pi \int_a^b R^2 - r^2 \, dr$

$$V(x) = \pi \int_{.24}^{1} (1 - e^{-3x})^2 - (1 - x^{1/2})^2 \, dx = 1.423 \text{ units}^3$$

Work for problem 1(c)

$\sqrt{x} - e^{-3x}$ = length of base

$5(\sqrt{x} - e^{-3x})$ = height

Area of Rec = base x height

$A = 5(\sqrt{x} - e^{-3x})^2$

To find Volume, integrate the area - use disc method

$$V = 5\pi \int_{.24}^{1} (\sqrt{x} - e^{-3x})^2 \, dx = 6.953 \text{ units}^3$$

GO ON TO THE NEXT PAGE.

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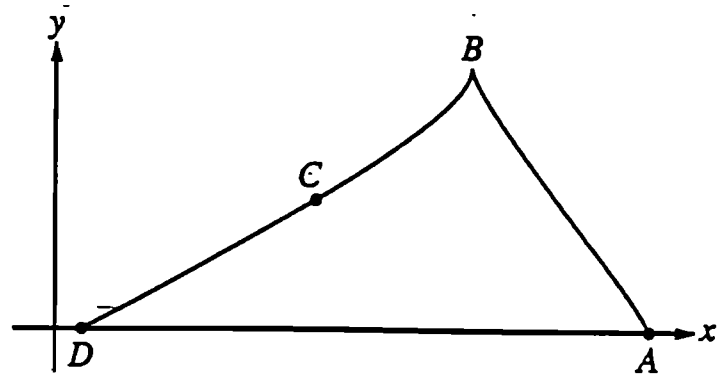


Work for problem 2(a)

At C both $\frac{dy}{dt}$ and $\frac{dx}{dt}$ are negative since the particle is traveling in the negative x and y directions. Each component of the velocity vector must be negative for the particle to travel down to the left.

Work for problem 2(b)

$t>0$

$-9\cos\frac{\pi t}{6}\sin\left(\frac{\pi\sqrt{t+1}}{2}\right)=0$ $\quad \frac{\frac{dy}{dt}}{\frac{dx}{dt}}\to 0$

$\cos\frac{\pi t}{6}=0$ $\qquad \sin\frac{\pi\sqrt{t+1}}{2}=0$

$t=3,9\ldots$ $\qquad t=3\ldots$

at $t=3$

Continue problem 2 on page 7.

Work for problem 2(c)

$\frac{\frac{dy}{dt}}{\frac{dx}{dt}}(8) = \frac{5}{9}$

$\frac{\frac{dy}{dt}}{-4.5} = \frac{5}{-9}$

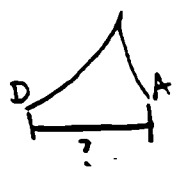
$\frac{dy}{dt} = -2.5$

$\vec{v} = \langle x'(t), y'(t) \rangle$

$\vec{v}(8) = \langle -4.5, -2.5 \rangle$

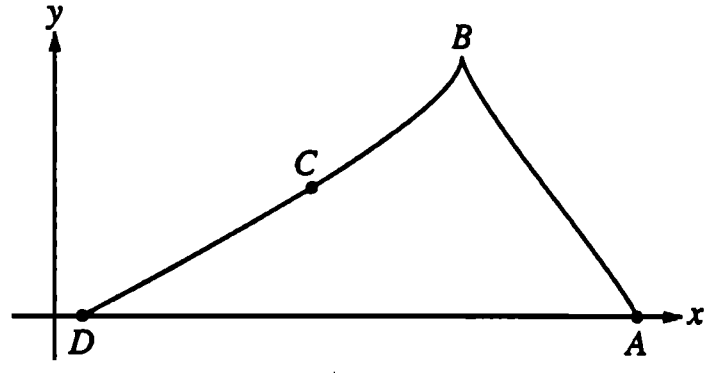
speed $= |\vec{v}|_8 = \sqrt{(-4.5)^2 + (-2.5)^2} = 5.148$

Work for problem 2(d)



$dist = \left| \int_0^9 \left[9 \cos\frac{\pi t}{6} \sin\frac{\pi\sqrt{t+1}}{2} \right] dt \right| = |-39.255| = 39.255$

GO ON TO THE NEXT PAGE.



Work for problem 2(a)

$\frac{dx}{dt} = -9\cos\left(\frac{\pi t}{6}\right)\sin\left(\pi\frac{\sqrt{t+1}}{2}\right)$

at part C $\frac{dy}{dt}$ is negative. this is because the particle is going downward at this point.

at pt C $\frac{dx}{dt}$ is negative. this is because the particle is moving to the left at this point.

Work for problem 2(b)

pt B: $\frac{dx}{dt} = 0$

$-9\cos\left(\frac{\pi t}{6}\right)\sin\left(\pi\frac{\sqrt{t+1}}{2}\right) = 0$

when $t = 3$, $\frac{dx}{dt} = 0$

so pt B is $t = 3$

Continue problem 2 on page 7.

Work for problem 2(c)

$(x(8), y(8))$ $y = \frac{5}{9}x - 2$

$x'(8) = -9\cos\left(\frac{\pi 8}{6}\right)\sin\left(\pi\sqrt{\frac{8+1}{2}}\right)$

$= -9\cos\left(\frac{4\pi}{3}\right)\sin\left(\frac{3\pi}{2}\right)$

$= -4.5$

$dx = 4.5$

$\frac{dy}{dx} = \frac{5}{9}$

$dy = \frac{5}{9} \cdot \frac{9}{2}$

$\left(-\frac{9}{2}, -\frac{10}{8}\right)$

Work for problem 2(d)

$\text{distance} = \int_0^9 -9\cos\left(\frac{\pi t}{6}\right)\sin\left(\pi\sqrt{\frac{t+1}{2}}\right)$

$= -39.255$

point A and D are 39.255 units apart

GO ON TO THE NEXT PAGE.

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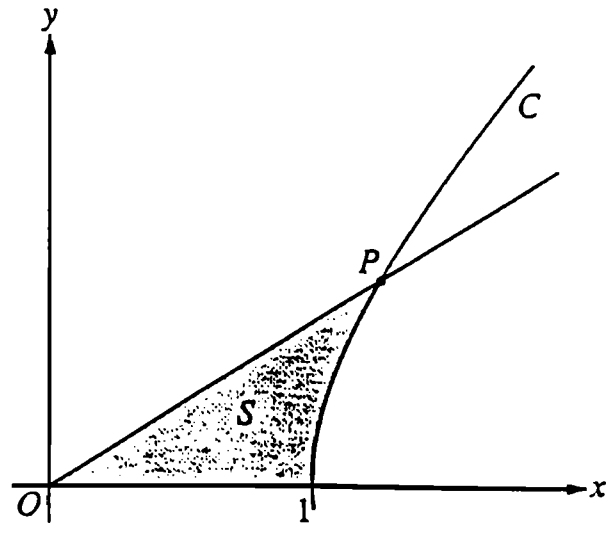


Work for problem 3(a)

$5/3\, y = \sqrt{1+y^2}$

$y = .75$

$x = 5/3\,(.75)$

$= 1.25$

P is at $(1.25, .75)$

$dx/dy = 1/2\,(1+y^2)^{-1/2} \cdot 2y$ $\qquad dx/dy\big|_{y=.75} = .6$

$= \dfrac{y}{\sqrt{1+y^2}}$

Work for problem 3(b)

$A = -\int_0^{.75} 5/3\, y\, dy + \int_0^{.75} \sqrt{1+y^2}\, dy$

$= \int_0^{.75} \left(\sqrt{1+y^2} - 5/3\, y\right) dy$

$\approx .347$

Continue problem 3 on page 9.

3 3 3 3 3 3 3 3 3 3 Q_2

Work for problem 3(c)

$x = r\cos\theta \qquad y = r\sin\theta$

$x^2 - y^2 = 1$

$(r\cos\theta)^2 - (r\sin\theta)^2 = 1$

$r^2\cos^2\theta - r^2\sin^2\theta = 1$

$r^2(\cos^2\theta - \sin^2\theta) = 1$

$$r^2 = \frac{1}{\cos^2\theta - \sin^2\theta}$$

Work for problem 3(d)

$\tan\theta = y/x$

$\tan\theta = \frac{.75}{1.25}$

$\theta \approx .540$

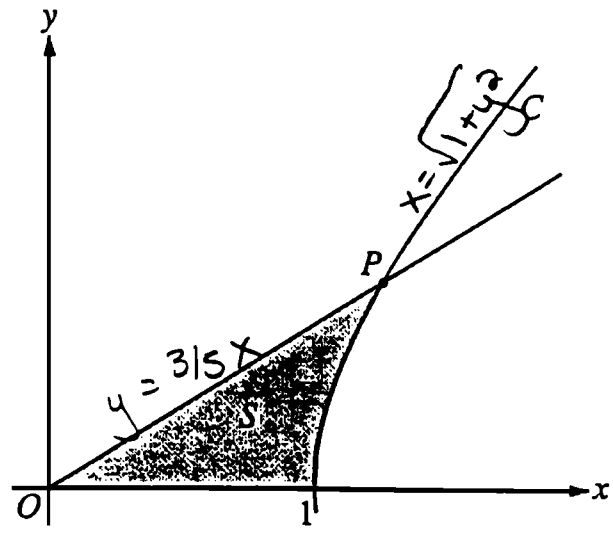
$$A = \frac{1}{2}\int_0^{.540} \frac{d\theta}{\cos^2\theta - \sin^2\theta}$$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

$x = \frac{5}{3}y$

$y = \frac{3}{5}x$



Work for problem 3(a)

$\frac{5}{3}y = \sqrt{1+y^2}$

when $y = .75$

$\left(\frac{5}{3}\right)(.75) = 1.25$

$P = (1.25, .75)$

$x = (1+y^2)^{1/2}$

$\frac{dx}{dx} = \frac{1}{2}(1+y^2)^{-1/2}\left(2y\frac{dy}{dx}\right)$

$1 = \frac{y\frac{dy}{dx}}{\sqrt{1+y^2}}$

$\frac{\sqrt{1+y^2}}{y} = \frac{dy}{dx}$ $\quad \frac{\sqrt{1+(.75^2)}}{.75} = \frac{5}{3}$

Work for problem 3(b)

$\int_0^{.75}\left(\left(\sqrt{1+y^2}\right) - \left(\frac{5}{3}y\right)\right)dy = S$

$S = .3466$

Continue problem 3 on page 9.

Work for problem 3(c)

$y = r\sin\theta$

$x = r\cos\theta$

$y^2 = r^2\sin^2\theta$

$x^2 = r^2\cos^2\theta$

$x^2 - y^2 = 1$

$r^2\cos^2\theta - r^2\sin^2\theta = 1$

$r^2(\cos^2\theta - \sin^2\theta) = 1$

$r^2 = \dfrac{1}{\cos^2\theta - \sin^2\theta}$

Work for problem 3(d)

$$\frac{1}{2}\int_0^{.6545} \frac{1}{\cos^2\theta - \sin^2\theta}\, d\theta$$

END OF PART A OF SECTION II

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4 4 4 4 4 4 4 4 4 4

NO CALCULATOR ALLOWED

C1

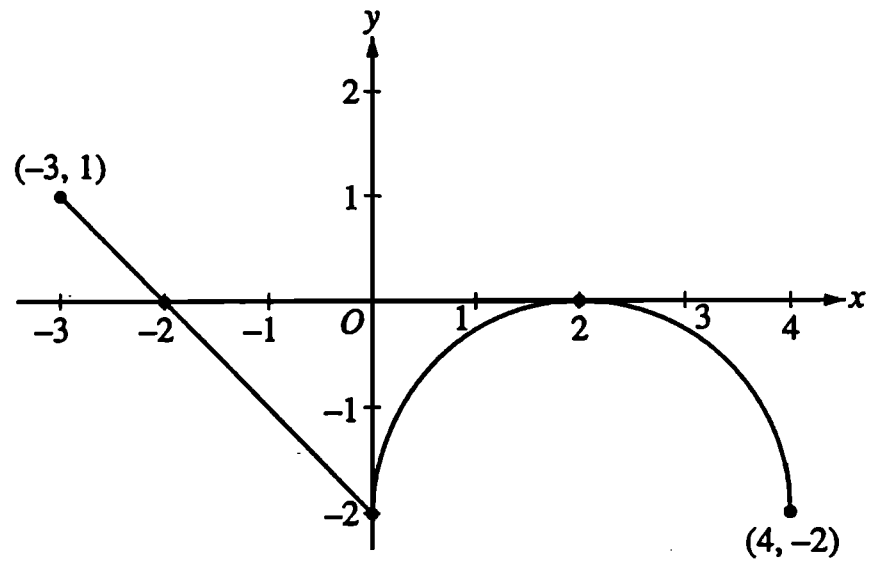
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f'

Work for problem 4(a)

$-3 < x < -2$; $f'(x)$ is positive

Work for problem 4(b)

$x = 0, 2$;

x	$-3<x<0$	0	$0<x<2$	2	$2<x<4$
$f''(x)$	−	undefined	+	0	−

Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

C_2

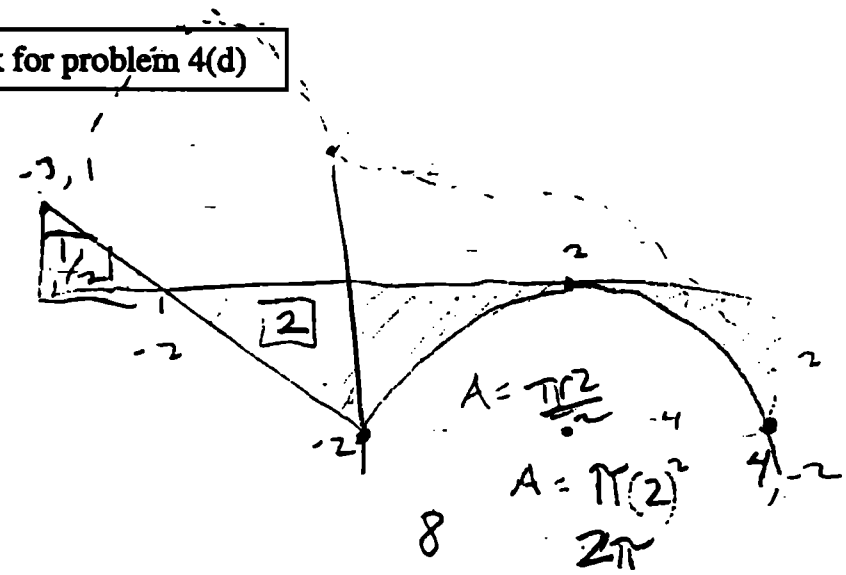
Work for problem 4(c)

point: $(0, 3)$

$f'(0) = -2$

$y - 3 = -2(x - 0)$

Work for problem 4(d)



$f(-3) = \int_0^{-3} f'(x)\,dx + 3$

$2 - \frac{1}{2} + 3 = 4.5$

$f(4) = \int_0^{4} f'(x)\,dx + 3$

$-(8 - 2\pi) + 3$

$-8 + 2\pi + 3$

$2\pi - 5$

GO ON TO THE NEXT PAGE.

4 4 4 4 4 4 4 4 4 4 E1

NO CALCULATOR ALLOWED

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

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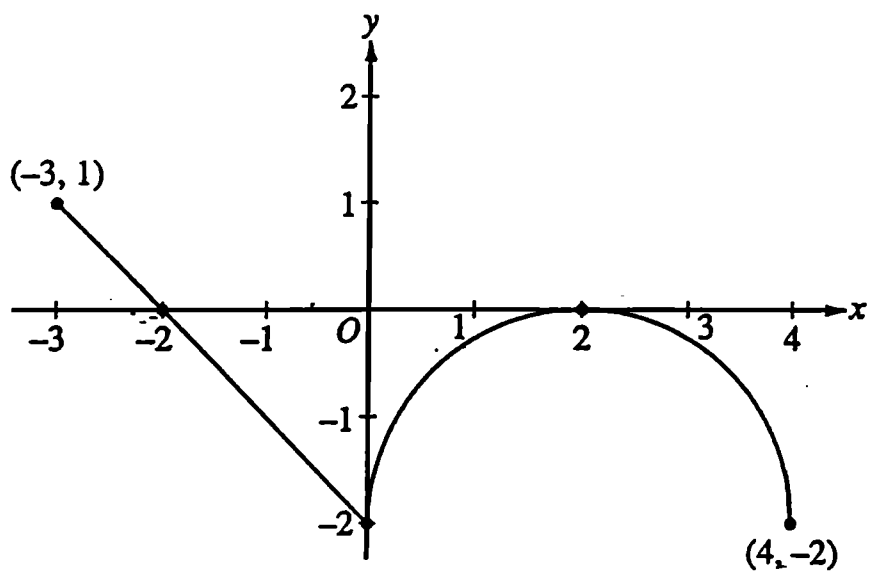

Graph of f'

Work for problem 4(a)

Increasing from $x=-3$ to $x=-2$

because the derivative is positive

Work for problem 4(b)

$x=0$ and $x=2$

these are the local maxima and minima of $f'(x)$

Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

E_2

Work for problem 4(c)

at $(0,3)$ slope is -2

$y-3=-2(x-0)$

$y=-2x+3$

Work for problem 4(d)

$f(0)=3$

$\int_0^{-3} f'(x) = -2+.5$ or -1.5

$3-1.5$

so $f(-3)=1.5$

$\int_0^4 f'(x) = -\frac{1}{2}\pi(2)^2 = -2\pi$

$f(0)-2\pi = f(4)$

$f(4)=3-2\pi$

GO ON TO THE NEXT PAGE.

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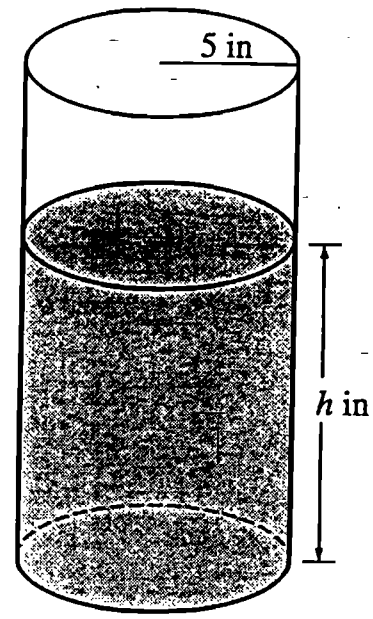


Work for problem 5(a)

$V = \pi r^2 h$

$\frac{dV}{dt} = -5\pi\sqrt{h} = \pi 25 \frac{dh}{dt}$

$\downarrow$

$-\frac{5\sqrt{h}}{25} = 25\frac{dh}{dt} \div 25$

$-\frac{\sqrt{h}}{5} = \frac{dh}{dt}$

Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(b)

$\int dh\, h^{-1/2} = \int -\frac{1}{5}\, dt$

$\int h^{-1/2}\, dh = \int -\frac{1}{5}\, dt$

$2h^{1/2} + C_1 = -\frac{1}{5}t + C_2$

$2\sqrt{h} = -\frac{1}{5}t + C_3$

$2\sqrt{17} = -\frac{1}{5}(0) + C_3$

$C_3 = 2\sqrt{17}$

$\frac{2\sqrt{h}}{2} = \frac{-\frac{1}{5}t + 2\sqrt{17}}{2}$

$\sqrt{h} = -\frac{t}{10} + \sqrt{17}$

$h = \left(-\frac{t}{10} + \sqrt{17}\right)^2$

Work for problem 5(c)

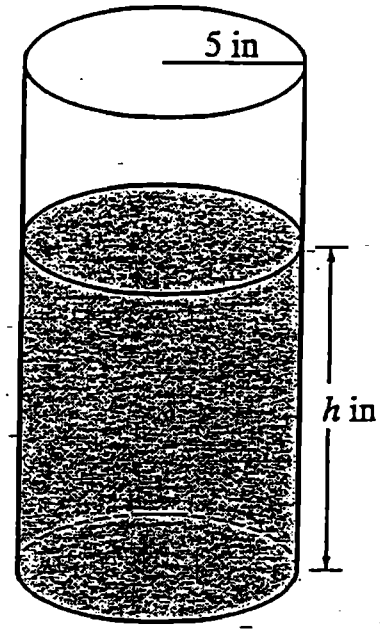
$h = 0$

$0 = \left(-\frac{t}{10} + \sqrt{17}\right)\left(-\frac{t}{10} + \sqrt{17}\right)$

$\frac{t}{10} = \sqrt{17}$

$t = 10\sqrt{17}$ seconds

NO CALCULATOR ALLOWED



- h = depth in inches

$\frac{dV}{dt} = -5\pi\sqrt{h}$ in³/s

$V = \pi r^2 h$

$V = \pi(25)h$

Work for problem 5(a)

$V = \pi r^2 h \quad r = 5$

$V = 25\pi h$

$\frac{dV}{dt} = 25\pi \frac{dh}{dt}$

$\frac{-5\pi\sqrt{h}}{25\pi} = \frac{25\pi}{25\pi} \cdot \frac{dh}{dt}$

$\frac{-\sqrt{h}}{5} = \frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$

$\frac{dV}{dt} = -5\pi\sqrt{h}$

Continue problem 5 on page 13.

Work for problem 5(b)

$$\frac{dh}{dt} = \frac{-\sqrt{h}}{5}$$

$$\frac{1}{\sqrt{h}}\,dh = -\frac{\sqrt{h}}{5}\,dt \cdot \frac{1}{\sqrt{h}}$$

$$\int h^{-1/2}\,dh = \int -\frac{1}{5}\,dt$$

$$2h^{1/2} = -\frac{1}{5}t + C$$

$$2\sqrt{h} = -\frac{1}{5}t + C \qquad h = 17 \text{ at time } t = 0$$

$$2\sqrt{17} = -\frac{1}{5}(0) + C$$

$$C = 2\sqrt{17}$$

$$\frac{2\sqrt{h}}{2} = \frac{\frac{1}{5}t + 2\sqrt{17}}{2}$$

$$(\sqrt{h})^2 = \left(\frac{1}{10}t + \sqrt{17}\right)^2$$

$$h = \left(\frac{t}{10} + \sqrt{17}\right)^2$$

Work for problem 5(c)

$V = 0 = 2\sqrt{h}$

$h = 0$

$$0 = \left(\frac{t}{10} + \sqrt{17}\right)^2$$

$$0 = \frac{t}{10} + \sqrt{17}$$

$$-10 \cdot -\sqrt{17} = \frac{t}{10} \cdot 10$$

$t = 10\sqrt{17}$ seconds

GO ON TO THE NEXT PAGE.

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NO CALCULATOR ALLOWED

Work for problem 6(a)

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!} = 1 = \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots$$

$$f'(x) = -\frac{2x}{3!} + \frac{4x^3}{5!} - \dots$$

$$f''(x) = -\frac{2}{3!} + \frac{4 \times 3 x^2}{5!} - \dots$$

$f'(0) = 0$

$f''(0) = -\frac{2}{3!} = -\frac{1}{3}$ Second derivative test

f has a local maximum at $x=0$ because $x=0$ is a critical point and f is concave down at that point

Work for problem 6(b)

Alternating Series

Error must be less than next term.

$$\text{Error} \le \frac{1}{5!} = \frac{1}{5 \times 4 \times 3 \times 2} = \frac{1}{20 \times 6} = \frac{1}{120}$$

$$\text{Error} \le \frac{1}{120} < \frac{1}{100}$$

Thus $1 - \frac{1}{3!}$ approximates $f(1)$ with an error less than $\frac{1}{100}$

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$$f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n+1)!}$$

$$f'(x)=\sum_{n=0}^{\infty}\frac{2n(-1)^n x^{2n-1}}{(2n+1)!}$$

$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$$

$$xy'+y = x\sum_{n=0}^{\infty}\frac{2n(-1)^n x^{2n-1}}{(2n+1)!}+\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n+1)!}$$

$$=\sum_{n=0}^{\infty}\frac{2n(-1)^n x^{2n}}{(2n+1)!}+\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n+1)!}$$

$$=\sum_{n=0}^{\infty}\frac{2n(-1)^n x^{2n}+(-1)^n x^{2n}}{(2n+1)!}=\sum_{n=0}^{\infty}\frac{(2n+1)(-1)^n x^{2n}}{(2n+1)(2n)!}$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}=\cos x$$

Thus $y=f(x)$ is a solution to the differential equation

$$xy'+y=\cos x$$

END OF EXAMINATION

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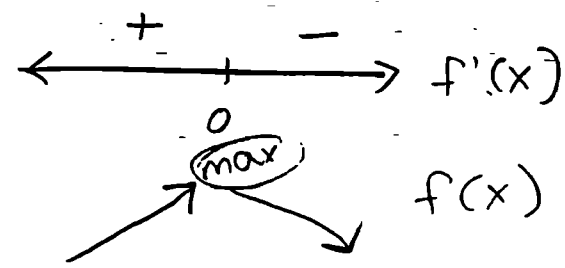
NO CALCULATOR ALLOWED

Work for problem 6(a)

$$f(x) = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \dots$$

$$f'(x) = -\frac{2x}{3!} + \frac{4x^3}{5!} - \frac{6x^5}{7!} + \dots$$

$$f'(0) = 0$$



$$f''(x) = -\frac{2}{3!} + \frac{12x^2}{5!} - \frac{30x^4}{7!} + \dots$$

$$f''(0) = -\frac{2}{3!} = -\frac{1}{3}$$

local max @ $x = 0$

$$f''(0) = -\frac{1}{3}$$

Work for problem 6(b)

Series is alternating so

error < first neglected term

first neglected term = $\frac{x^4}{5!}$

$$@1 = \frac{1}{5!} = \frac{1}{120}$$

$$\frac{1}{120} < \frac{1}{100}$$

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$$xy' + y = \cos x$$

$$xy' = \cos x - y$$

$$y' = \frac{\cos x - y}{x}$$

$$\frac{dy}{dx} = \frac{\cos x - y}{x}$$

$$-\frac{\cos x}{x} + \frac{dy}{dx} = -\frac{y}{x}$$

$$\int -\frac{\cos x}{x} + \frac{dy}{y} = \int -\frac{1}{x}\,dx$$

$$-\frac{\cos x}{x} + \int \frac{dy}{y} = \int -\frac{dx}{x}$$

$$+ \ln|y| + C_1 = -\ln|x| + C_2$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

$$x \cdot \left(-\frac{2x}{3!} + \frac{4x^3}{5!} - \frac{6x^5}{7!} + \dots\right) + \left(1 - \frac{x^2}{3!} + \frac{x^4}{5!}\right)$$

$$-\frac{2x^2}{3!} + \frac{4x^4}{5!} - \frac{6x^6}{7!} + \dots + 1 - \frac{x^2}{3!} + \frac{x^4}{5!} -$$

$$1 - \left(\frac{3x^2}{3!}\right) + \left(\frac{5x^4}{5!}\right) - \dots$$

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots = \cos x$$

END OF EXAMINATION

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